This is taking abstract 1, and i had a few questions that i was hoping you could help me with. They are as follows:

1. let G be an internal direct product of H,K. Show that hk=kh, for any k in K, h in H.

   Done in class

   Let $h \in H, k \in K$ then $hk = kh$ iff $hk(kh)^{-1} = hkh^{-1}k^{-1}$ iff $hkh^{-1}k^{-1} \in H \bigcap K$

   Since H normal subgroup so $kh^{-1}k^{-1} \in H$ so $hkh^{-1}k^{-1} \in H$. Similarly, since K normal subgroup so $hkh^{-1} \in K$ so $hkh^{-1}k^{-1} \in K$ and so $hkh^{-1}k^{-1} \in H \bigcap K$

2. Let G be a group and let H be a unique subgroup of order lHl. show that H is a normal subgroup of G.

   For any $x \in G$, $|xHx^{-1}| = |H|$ and so for any $x \in G$, $xHx^{-1} = H$

3. let G be a group of order pq, where p and q are distinct primes. Show that if $Z(G) \neq \{e\}$, then G is cyclic.

   if $Z(G) \neq \{e\}$ then $|Z(G)| = p, q$ or pq. If $|Z(G)| = p$ or q then $|G/Z(G)|$ is prime and so $G/Z(G)$ is cyclic and so G is abelian and if $|Z(G)| = pq$ then G is abelian and so in all cases G is abelian and by Cauchy's theorem, G has elements a, b of order p, q respectively, and so ab is a generator of G

4. Let H be a subgroup of G if G/H is a group, show that H is normal in G.

   Let $x \in G$, so $xHx^{-1} = xHHx^{-1} = xx^{-1}H = H$ and so $H \trianglelefteq G$

5. If H is normal in G and K is a subgroup of G, then H intersection K is a normal subgroup of K. First $H \bigcap K$ is a subgroup of G (prove it)

   Now let $k \in K$ and $x \in H \bigcap K$, then $kxk^{-1} \in H$ by normality and $kxk^{-1} \in K$ by closure and so $kxk^{-1} \in H \bigcap K$

6. let $f : G \to H$ is a group Homomorphism . Show that f is 1-1 if kerf=e

   Let $kerf = \{e\}$ and let $x, y \in G$ such that $f(x) = f(y)$ and so $f(x)f(y)^{-1} = f(xy^{-1}) = e \in kerf$ and so $xy^{-1} = e$, and so $x = y$